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Three-dimensional Heisenberg spin-glass models with and without random anisotropy

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Abstract

We re-examine the spin-glass (SG) phase transition of the $\pm J$ Heisenberg models with and without random anisotropy D in three dimensions ($d = 3$) using two complementary methods, i.e., (i) the defect energy method and (ii) the Monte Carlo method. We reveal that the conventional defect energy method is not convincing and propose a new method which considers the stiffness of the lattice itself. Using the method, we show that the stiffness exponent θ has a positive value ($\theta > 0$) even when $D = 0$. Considering the stiffness at finite temperatures, we obtain the SG phase transition temperature of $T_{SG} \sim 0.19J$ for $D = 0$. On the other hand, a large scale MC simulation shows that, in contrast to the previous results, a scaling plot of the SG susceptibility χ_{SG} for $D = 0$ is obtained using almost the same transition temperature of $T_{SG} \sim 0.18J$. Hence we believe that the SG phase transition occurs in the Heisenberg SG model in $d = 3$.

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1. Introduction

For a long time, it has been believed that the spin-glass (SG) phase is realized in three dimensions ($d = 3$) for the Ising model [1, 2] but not for the XY and Heisenberg models [3–7]. Thus, the SG phases observed in experiments were suggested to be realized due to anisotropy [8, 9]. However, numerical studies in the last decade have revealed that the SG phase might be more stable than has been believed so far. In a long-range Ruderman–Kittel–Kasuya–Yoshida (RKKY) model, it was shown that the SG susceptibility exhibits a divergent singularity at a finite temperature, even when the anisotropy is absent [10, 11]. This behaviour has been attributed to the randomness of the spin position (site random model) rather than the long-range nature of the RKKY interaction. In fact, a short-range site random model composed of ferromagnetic spins and antiferromagnetic spins was shown to exhibit a long-range order phase characterized by the coexistence of a ferromagnetic and an antiferromagnetic order [12].

On the other hand, for the XY and Heisenberg bond SG models, Kawamura and his co-workers took note of the chiralities of the spins and showed that a chiral glass (CG) phase transition occurs at a finite temperature $T_{\mathrm{CG}} \neq 0$, but the spin-glass phase is still absent [13–15]. They insisted that an anisotropy mixes the chiral freedom and the spin freedom and the SG phase transition occurs at $T_{\mathrm{SG}} (= T_{\mathrm{CG}})$. This view of the SG phase transition is quite attractive, because it gives a novel picture of the SG phase. That is, in the picture, the CG phase realizes in the real world, not the SG phase. However, their bases for the absence of the SG phase are obscure. Moreover, since the chirality is described by the spin variables, then the origin of the CG phase transition might be the usual SG phase transition. In fact, recent studies of the ageing effects of the spin and the chirality autocorrelation functions [16] and the developments of the SG and the CG susceptibilities [17] by means of a nonequilibrium relaxation method suggested that, if the CG phase transition occurs, the SG phase transition occurs at the same transition temperature $T_{\mathrm{SG}} = T_{\mathrm{CG}}$. Quite recently, Lee and Young presented the same conclusion using a finite-size analysis of the correlation length of the spins and chiralities [18].

During the last decade, new algorithms for simulating complex systems have been developed and available computer power has increased enormously. It is therefore possible to re-examine in detail the SG phase transition of the Heisenberg model on the basis of the usual analyses. Here we consider Heisenberg models with and without random anisotropy on a simple cubic lattice described by

$$H = -\sum_{\langle ij \rangle} \left[J_{ij} \boldsymbol{S}_i \boldsymbol{S}_j + \sum_{\alpha \neq \beta} D_{ij}^{\alpha\beta} S_i^{\alpha} S_j^{\beta} \right] \tag{1}$$

where $\boldsymbol{S}_i$ is the Heisenberg spin of $|\boldsymbol{S}_i| = 1$ and S_i^{α} is its α-component ($\alpha = x, y, z$), and $\langle ij \rangle$ runs over all nearest-neighbour pairs. The exchange interaction J_{ij} takes on either $+J$ or $-J$ with the same probability of $\frac{1}{2}$. We assume that the anisotropy comes from pseudo-dipolar couplings and impose the restriction $D_{ij}^{\alpha\beta} = D_{ji}^{\alpha\beta} = D_{ij}^{\beta\alpha}$. We further assume that $D_{ij}^{\alpha\beta}$ are uniform random values between $-D$ and D.

Evidence for the absence of the SG phase in the Heisenberg SG model which has been believed so far is given by the following two points.

(i) Negative stiffness exponent θ at $T = 0$ [3, 4, 13].
(ii) Scaling plots of the SG susceptibility χ_{SG} and absence of the crossing of the Binder ratio g_L [5, 19].

Then we re-examine these two points to consider the possibility of the SG phase transition of the Heisenberg SG model. We will consider stiffness exponent θ at $T = 0$ and $T \neq 0$ in section 2, and properties of χ_{SG} and g_L in section 3. We will give special attention to the effect of the anisotropy, because it has been believed that the anisotropy brings about the SG phase transition. So if it is true, we will find different properties between models with and without anisotropy.

2. Stiffness exponent θ

The most accepted evidence for the absence of the SG phase is the results of the defect energy method. So we first consider the defect energy method.

2.1. Conventional defect energy method

The defect energy method comes from an application of a renormalization-group idea [3, 4, 20]. That is, one evaluates the effective coupling $\tilde{J}_L$ between block spins of the

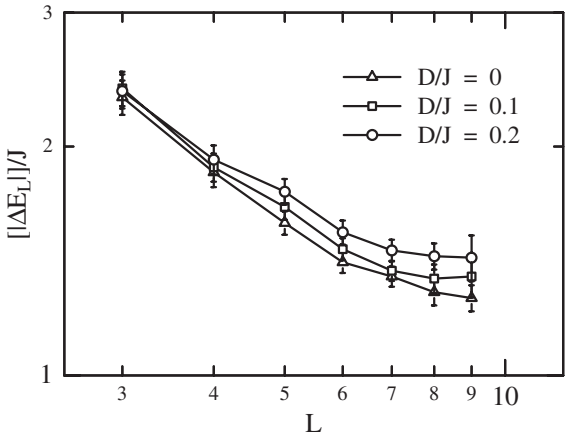

Figure 1. The lattice size dependence of the naive defect energy $[|\Delta E_L|]$ of the Heisenberg SG model in $d = 3$.

linear dimension L generated by the renormalization. The effective coupling $\tilde{J}_L$ would depend on L as $\tilde{J}_L \sim JL^{\theta}$ with θ being called the stiffness exponent. When $\theta > 0$, the SG phase transition occurs at a finite temperature, while no phase transition occurs at any finite temperature when $\theta < 0$. To estimate $\tilde{J}_L$, one considers the domain-wall energy ΔE_L which is defined as the difference in the ground-state energy of two lattices A and B of size $L \times L \times L$ with the same bond distribution but with different boundary conditions. That is, for lattice A, a periodic boundary condition is applied for every direction, and, for lattice B, an antiperiodic boundary condition is applied for one direction and the periodic boundary condition for the other directions. By using this method, Banavar and Cieplak first estimated the value of $\theta \sim -1$ and predicted that the SG phase transition occurs at $T = 0$ [3]. Successive studies also predicted negative values for θ, i.e., $\theta \sim -0.65$ [4] and $\theta \sim -0.49$ [13].

Recently, however, a doubt was thrown on the estimation of $\tilde{J}_L$ [21, 22]. That is, in the calculation of ΔE_L, one expects that no domain wall exists in the lattice A (or B) and hence one domain wall arises in the lattice B (or A). This expectation might be true, but another possibility would be equally true. That is, some domain wall will occur in the lattice A and some different domain wall in the lattice B. Then one might examine merely the difference in energy between those two domain walls. Does this defect energy ΔE_L really give the effective coupling $\tilde{J}_L$ between the block spins? So we first examine ΔE_L of the model with and without random anisotropy. We calculate ΔE_L for lattices of $L \leqslant 9$, and for each L, the sample averages are taken over about 4000 independent bond realizations. Results of $[|\Delta E_L|]$ are presented in figure 1 in log–log form, where $[\cdots]$ means the sample average. Data for $D = 0$ are curved. The most surprising thing is that this L-dependence of $[|\Delta E_L|]$ is similar to those in the case of $D \neq 0$. These results suggest two possibilities. One is that the finite-size effect is so large that the asymptotic region has not yet been reached. Since the curvature is upwards, it is possible that $\theta \sim 0$ or even $\theta > 0$ in the limit of $L \to \infty$. The other is the inadequateness of estimating the defect energy as pointed out above. In order to examine the latter possibility, we study this problem in a different method.

2.2. Stiffness of the system

Apart from the renormalization-group concept, we consider the stability of the spin configuration of the system itself [21, 23]. The strategy of our examination is as follows

[21, 24, 25, 27]. We prepare a cubic lattice of $L \times L \times (L+1)$ with an open boundary condition in one direction of $(L+1)$ lattice sites (z-direction) and the periodic boundary condition in the other directions. That is, the lattice has two surfaces Ω_1 and Ω_{L+1}. We call this system the reference system. First we determine the ground-state spin configuration of the reference system. Hereafter, the ground-state spin configurations on Ω_1 and Ω_{L+1} are denoted as $\{\boldsymbol{S}_i{}^{(1)}\}$ and $\{\boldsymbol{S}_i{}^{(L+1)}\}$, respectively. In this spin configuration, any distortion (domain wall) in the z-direction will be removed, because the lattice has free surfaces Ω_1 and Ω_{L+1}. Then we add a distortion inside the system in such a manner that $\{\boldsymbol{S}_i{}^{(1)}\}$ are fixed and $\{\boldsymbol{S}_i{}^{(L+1)}\}$ are changed under the condition that the relative angles between the spins are fixed. The ground-state energy of this system is always higher than that of the reference system. When $D = 0$, this excess energy is the net energy added inside the reference system, because the surface energy of Ω_{L+1}, which is given as the sum of the exchange energies between the spins on Ω_{L+1}, is conserved. We consider the stability of the system on the basis of this excess energy. One might think that the fixing of the relative spin directions on Ω_1 and Ω_{L+1} overestimates the stability of the spin configuration. We think, however, that this restriction is not serious for discussing the stability, because the increase of the excess energy to infinity for $L \to \infty$ means nothing but the existence of a strong correlation between the spin configurations on Ω_1 and Ω_{L+1}. In fact, the same method was successfully applied to the Ising SG model in $d = 2$ [21, 25].

We calculate two kinds of excess energies. One is the excess energy which is gained by rotating $\{\boldsymbol{S}_i{}^{(L+1)}\}$ by the same angle ϕ around some common axis (z-axis) and the other is the excess energy which is gained by reversing $\{\boldsymbol{S}_i{}^{(L+1)}\}$. Hereafter, we call the former system the rotated system and the latter system the reversed system. We think that it is sufficient to examine these two excess energies for considering the stiffness, because we can change $\{\boldsymbol{S}_i{}^{(L+1)}\}$ into any direction by combining the rotation and the reversal. The excess energy for the rotation $\Delta E_{\rm rot}(\phi)$ and that for the reversion $\Delta E_{\rm rev}$ is given as

$$\Delta E_{\rm rot}(\phi) = E_{\rm rot}(\phi) - E_G \tag{2}$$

$$\Delta E_{\rm rev} = E_{\rm rev} - E_G \tag{3}$$

where E_G is the ground-state energy of the reference system, and $E_{\rm rot}(\phi)$ and $E_{\rm rev}$ are the ground-state energies of the rotated system and reversed system, respectively. The lattice sizes studied here are $L = 3$–8 and, for each L, the sample averages are taken over about 1000 independent bond realizations.

In figure 2, we present the L-dependence of $[\Delta E_{\rm rot}(\pi/2)]$. Here we show data only for $D = 0$, because in the case of $D \neq 0$ we could hardly evaluate the net excess energy of $[\Delta E_{\rm rot}(\pi/2)]$.[3] We clearly see that the data increase with L. From the slope of the asymptotic line shown in the figure, we tentatively determine the value of the stiffness exponent as $\theta_{\rm rot} \sim 0.62$. That is, the SG phase would not be destroyed by a rotational perturbation.

In figure 3, we present L-dependences of $[\Delta E_{\rm rev}]$ for both $D = 0$ and $D \neq 0$. Data depend little on the value D. For each D, they seem to lie on a curve with a common positive slope of $\theta_{\rm rev} \sim 0.4$. Again, we get the view that the SG phase is stable at $T \neq 0$ for both $D \neq 0$ and $D = 0$.

Our results suggest $\theta_{\rm rev}, \theta_{\rm rot} > 0$ for both $D = 0$ and $D \neq 0$. However, their values are somewhat different from each other. It should be pointed out, however, that these values of $\theta_{\rm rev}$ and $\theta_{\rm rot}$ may vary for $L \to \infty$, because in the lattice size range studied here $[\Delta E_{\rm rot}(\pi/2)]$ is smaller than $[\Delta E_{\rm rev}]$ and the former increases more rapidly than the latter ($\theta_{\rm rot} > \theta_{\rm rev}$),

[3] When $D \neq 0$, the uniform rotation of $\{\boldsymbol{S}_i{}^{L+1}\}$ gives an additional excess energy of the surface Ω_{L+1}.

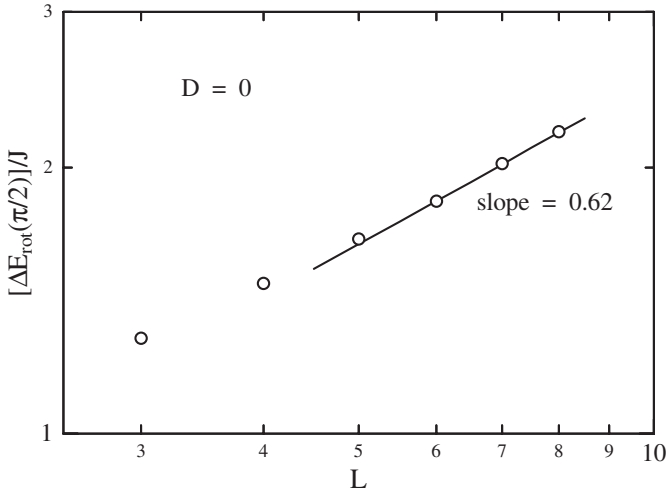

Figure 2. The lattice size dependence of the excess energy $[\Delta E_{\rm rot}(\pi/2)]$ of the Heisenberg SG model in $d = 3$.

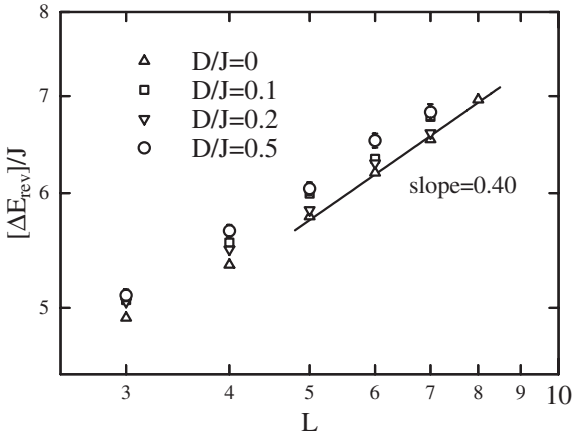

Figure 3. The lattice size dependence of the excess energy $[\Delta E_{\rm rev}]$ of the Heisenberg SG model in $d = 3$.

then as L increases further they would come close to each other. The convincing values of $\theta_{\rm rot}$ and $\theta_{\rm rev}$ would be given in that range of L. Unfortunately, the lattice sizes are still too small to examine this speculation. Anyway, both the analyses of $[\Delta E_{\rm rot}(\pi/2)]$ and $[\Delta E_{\rm rev}]$ suggest that the system tends to be *rigid* as the size of the lattice becomes larger. Note that we have also calculated the defect energies of the system for $D = 0$ using two replica boundary conditions [26] and found that they also increase with similar, positive slopes of $\theta_{\rm rot}^{\rm (rep)} \sim 0.59$ and $\theta_{\rm rev}^{\rm (rep)} \sim 0.46$ for the π rotation around the z-axis and the reversion, respectively [27]. Hence, we conclude that *the defect energy method never gives evidence of the phase transition at* $T_{\rm SG} = 0$.

What is the SG phase transition temperature $T_{\rm SG}$? We may estimate $T_{\rm SG}$ by calculating the excess free energy $[\Delta F_{\rm rot}(T)]$ and $[\Delta F_{\rm rev}(T)]$ [28] at finite temperatures. In fact, we have also calculated these quantities for $D = 0$ [27]. The result of $[\Delta F_{\rm rev}(T)]$ is shown in figure 4. It is seen that, at high temperatures, $[\Delta F_{\rm rev}(T)]$ decreases with increasing L, whereas at low

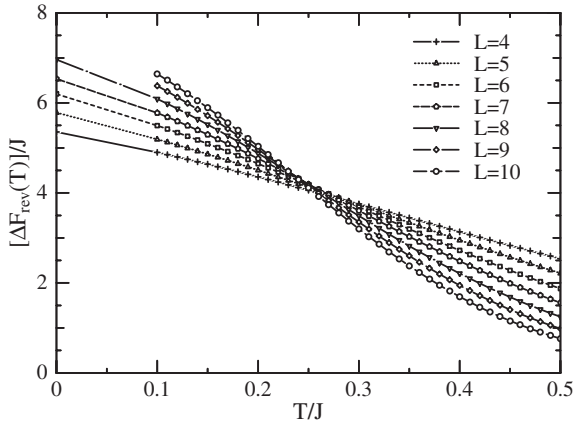

Figure 4. The lattice size dependence of the excess free energy $[\Delta F_{\rm rev}(T)]$ of the Heisenberg SG model for $D = 0$ in $d = 3$ [27]. Data at $T = 0$ are $[\Delta E_{\rm rev}]$ presented in figure 3.

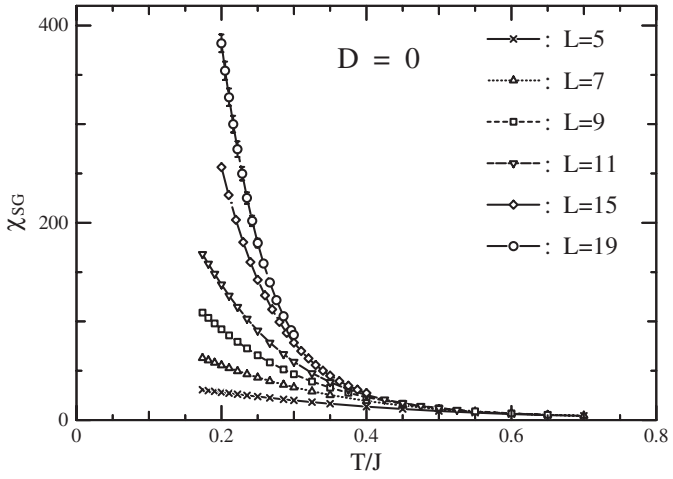

Figure 5. Temperature dependences of the spin-glass susceptibility $\chi_{\rm SG}$ of the $\pm J$ Heisenberg model in $d = 3$ for different sizes of the lattice at $D = 0$.

temperatures it increases with L. One estimates the phase transition temperature from the crossing temperature of the free energies for various lattice sizes L. In the present model, the crossing temperature T_L for the lattice sizes L and $L + 1$ shifts systematically to the low temperature side with increasing L. Then, we assumed that T_L decreases linearly with $1/L$, and estimated T_L for $L \to \infty$ as $T_\infty/J = 0.188 \pm 0.015$. Note that the same extrapolation for $[\Delta F_{\rm rot}(T)]$ gave $T_\infty/J = 0.192 \pm 0.015$. Therefore we may conclude that, if the SG phase transition occurs, the transition temperature is $T_{\rm SG} \sim 0.19J$.

3. Monte Carlo simulation

Now we re-examine the SG phase transition itself. Here we consider the model on a simple cubic lattice of $L \times L \times (L+1)(\equiv N)$ with skew boundary conditions along two L directions and a periodic boundary condition along the $(L + 1)$ direction. We perform a MC simulation

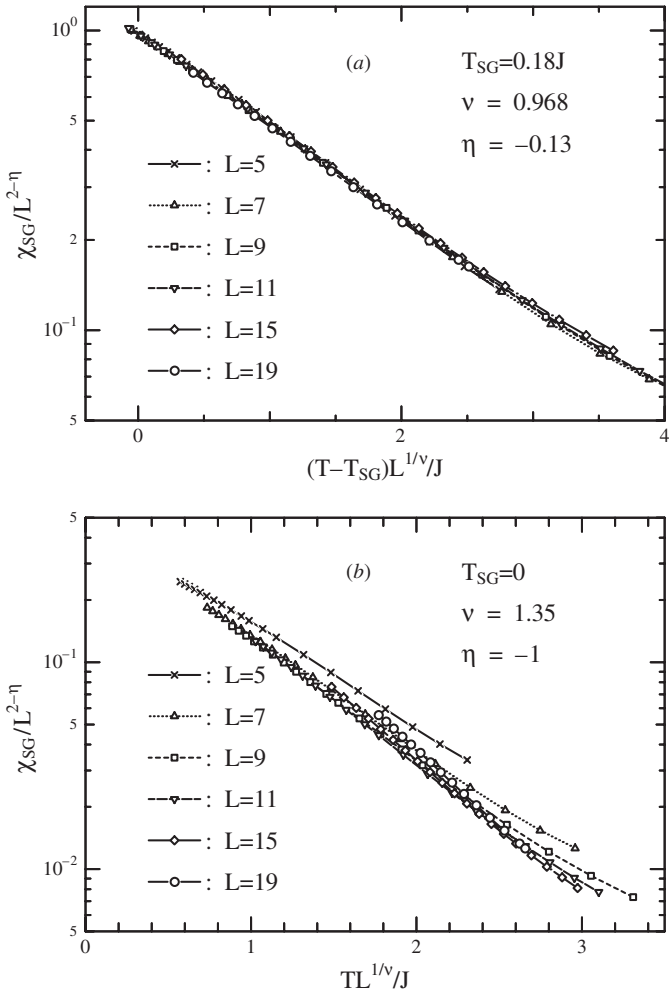

Figure 6. Typical examples of the finite-size scaling plot of the SG susceptibility at $D = 0$ for (*a*) $T_{\rm SG} \neq 0$ and (*b*) $T_{\rm SG} = 0$.

of the two-replica systems of the spins $\{\boldsymbol{S}_i\}$ and $\{\boldsymbol{T}_i\}$ using an exchange MC algorithm [29]. We calculate the order-parameter probability distribution $P_L(q)$ of

$$P_L(q) = [\langle \delta(q - Q) \rangle] \tag{4}$$

where $\langle \cdots \rangle$ and $[\cdots]$ mean the thermal average and the bond distribution average, respectively. Here Q is the spin overlap defined by

$$Q = \sqrt{\frac{1}{3} \sum_{\alpha,\beta} (q^{\alpha\beta})^2} \tag{5}$$

with $q^{\alpha\beta} \equiv \frac{1}{N} \sum_{i=1}^{N} S_i^{\alpha} T_i^{\beta}$. Using $P_L(q)$, we obtain two conventional SG quantities, i.e., the SG susceptibility $\chi_{\rm SG}$ and the Binder parameter $g(L, T)$ which are defined by

$$\chi_{\rm SG} = 3N[\langle q^2 \rangle] \tag{6}$$

$$g(L, T) = \frac{1}{2} \left(11 - 9 \frac{[\langle q^4 \rangle]}{[\langle q^2 \rangle]^2} \right) \tag{7}$$

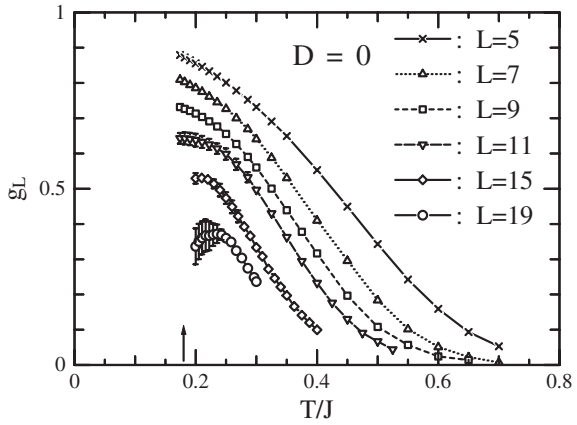

Figure 7. Temperature dependences of the Binder parameter $g(L, T)$ of the $\pm J$ Heisenberg model for $D = 0$ in $d = 3$ for different sizes of the lattice. The arrow indicates the transition temperature $T_{\rm SG}$ estimated from the scaling plot of $\chi_{\rm SG}$.

where $[\langle q^n \rangle] = \int q^n P_L(q)\,{\rm d}q$. We examine the size and temperature dependences of these quantities both for $D = 0$ and for $D \neq 0$. The linear sizes of the lattice studied here are $L = 5$–19. Equilibration is checked by monitoring the stability of the results against runs at least twice as long. The numbers of the samples are 480 for $L = 5$–9, 192 for $L = 11$, 96 for $L = 15$ and 48 for $L = 19$.

In figure 5, we show results for the SG susceptibility $\chi_{\rm SG}$ of the model without the anisotropy ($D = 0$). As the temperature is decreased, $\chi_{\rm SG}$ for larger L increases rapidly. If the lower critical dimension d_l is less than the lattice dimension, $d_l < 3$, and the phase transition really occurs at $T = T_{\rm SG}$, the data for different L will be scaled as

$$\chi_{\rm SG} = L^{2-\eta} F(L^{1/\nu}(T - T_{\rm SG})) \tag{8}$$

where ν is the exponent of the correlation length and η is the exponent which describes the decay of the correlation function at $T = T_{\rm SG}$. The scaling plots obtained by assuming $T_{\rm SG} \neq 0$ and $T_{\rm SG} = 0$ are shown in figure 6. The scaling with $T_{\rm SG} \neq 0$ works better than that with $T_{\rm SG} = 0$, even if the data for the smallest size $L = 5$ are ignored in the latter[4]. Note that in the previous scaling analysis [9], $T_{\rm SG} = 0$ was estimated using the data for the lattice of $L = 7$–15. Here, we use the data for a wider temperature range and add the data of the bigger lattice of $L = 19$. The phase transition temperature and the values of the critical exponent estimated here are $T_{\rm SG}/J = 0.18 \pm 0.01$, $\nu = 0.97 \pm 0.05$ and $\eta = -0.1 \pm 0.1$. We should emphasize that this value of $T_{\rm SG}$ is in good agreement with that estimated from the excess free energy of $T_{\rm SG}/J \sim 0.19$. It is noted, however, that the possibility of $T_{\rm SG} = 0$ is not ruled out from the scaling plot of figure 6(*b*), because in that case the temperature range of $T \gtrsim 0.2J$ would be out of a critical region [32]. As the anisotropy is added, the transition temperature increases with D, i.e., $T_{\rm SG}/J = 0.32 \pm 0.03$ for $D = 0.2J$, and $T_{\rm SG}/J = 0.65 \pm 0.05$ for $D = 1.0J$.

The Binder parameter $g(L, T)$ is the other quantity for examining the SG phase transition. It is believed that, if the SG phase transition occurs at $T_{\rm SG}$, $g(L, T)$ for different L cross at $T_{\rm SG}$. In contrast to our expectation, as shown in figure 7, they neither cross nor come together at $T_{\rm SG}$. This result seems to give the opposite view about the SG phase transition. However,

[4] In the case of $T_{\rm SG} = 0$, we have made the scaling plot assuming η to be an adjustable parameter and found that the quality of the scaling plot is almost the same in the range of $-1.0 \leqslant \eta \leqslant -0.9$.

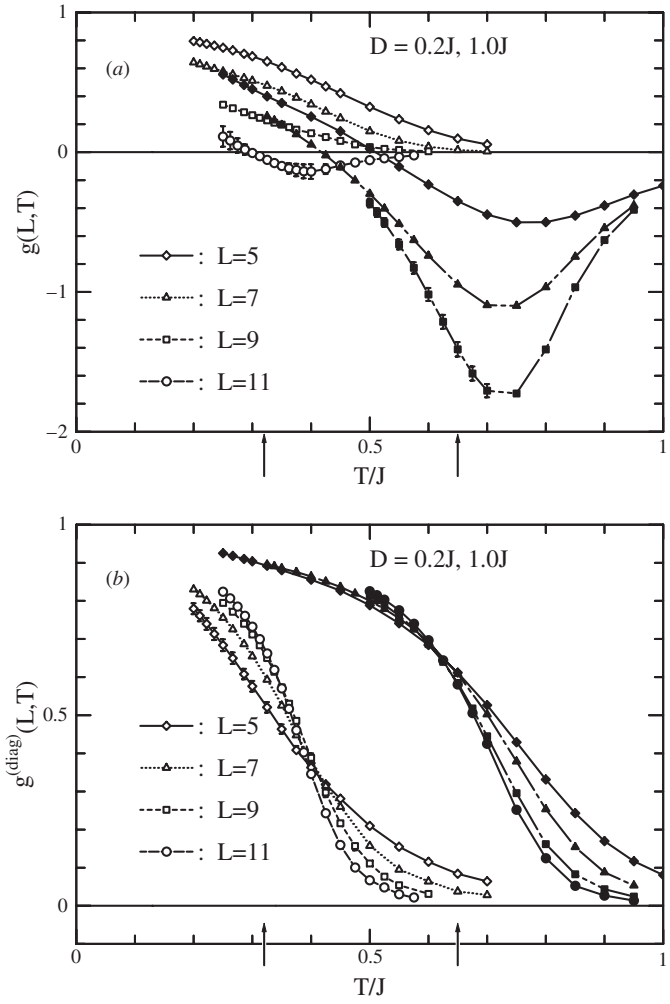

Figure 8. Temperature dependences of (*a*) the Binder parameter $g(L, T)$ and (*b*) the diagonal Binder parameter $g^{(\mathrm{diag})}(L, T)$ of the $\pm J$ Heisenberg model in $d = 3$ for different magnitudes of the anisotropy D and for different sizes of the lattice. Open symbols are for $D = 0.2J$ and closed ones for $D = 1.0J$. Arrows indicate the transition temperatures T_{SG} estimated from the scaling plot of χ_{SG}.

the absence of the crossing of $g(L, T)$ was also seen in the $\pm J$ Heisenberg model in four dimensions ($d = 4$) [30] in which the SG phase transition is believed to occur at some finite temperature even when $D = 0$ [7]. If the absence of the crossing of $g(L, T)$ for finite L says nothing about the SG phase transition, the same would be true when the anisotropy is present ($D \neq 0$). Then, we also calculate the Binder parameter of the model with $D \neq 0$. Here, since the system for $D \neq 0$ has inversion symmetry, we also consider the spin overlap of the diagonal components for which, in equation (4),

$$Q_{\mathrm{diag}} = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{S}_i \boldsymbol{T}_i \tag{9}$$

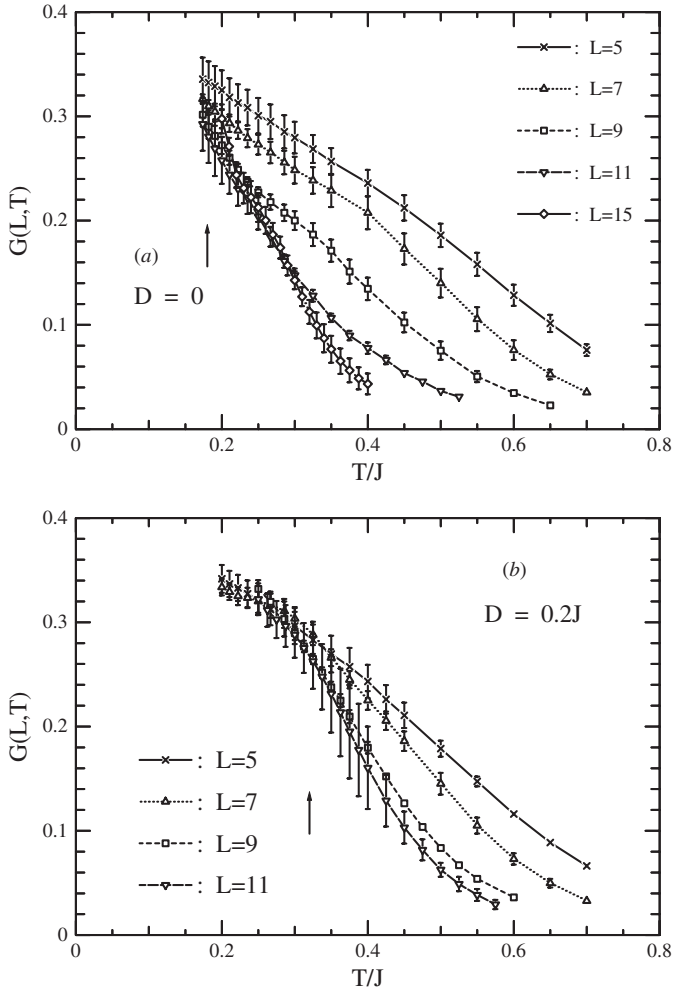

Figure 9. Temperature dependences of the order-parameter fluctuations $G(L, T)$ of the $\pm J$ Heisenberg model in $d = 3$ for different sizes of the lattice : (*a*) $D = 0$, and (*b*) $D = 0.2J$. The arrow indicates the transition temperature T_{SG} estimated from the scaling plot of χ_{SG}.

is used instead of Q. Hereafter, we call the SG susceptibility and the Binder parameter calculated using Q_{diag} the diagonal SG susceptibility and the diagonal Binder parameter and denote them as $\chi^{(\mathrm{diag})}$ and $g^{(\mathrm{diag})}(L, T)$, respectively. Results are presented in figures 8(*a*) and (*b*) for $g(L, T)$ and $g^{(\mathrm{diag})}(L, T)$, respectively. In fact, $g(L, T)$ for different L neither cross nor come together. In contrast, $g(L, T)$ for larger L exhibit a dip. As D is increased, this property becomes more prominent. In contrast $g^{(\mathrm{diag})}(L, T)$ exhibits *the usual behaviour*. That is, as the temperature is decreased, $g^{(\mathrm{diag})}(L, T)$ increases monotonically and its size dependence reverses. We suggest, hence, that the definition of the Binder parameter in terms of Q_{diag} is adequate for examining the phase transition for $D \neq 0$ and its crossing behaviour supports the presence of the phase transition. It is noted, however, that the crossing temperature seems to deviate considerably from that estimated above. We think that this deviation comes from a finite-size effect, because the crossing temperatures for different L exhibit a considerable L-dependence and, as L increases, it seems to approach T_{SG}. We have also calculated

$g^{(\mathrm{diag})}(L,T)$ in the case of $D=0$ and found the absence of crossing behaviour. We think this difference in the behaviour of $g^{(\mathrm{diag})}(L,T)$ comes from the occurrence of the drift of the whole system due to the $O(3)$ symmetry for $D=0$. In fact, the diagonal SG susceptibility $\chi_{\mathrm{SG}}^{(\mathrm{diag})}$ for $D=0$ has been found to be much smaller than χ_{SG}, whereas that for $D\neq 0$ is larger than χ_{SG}.[5] We believe, hence, that the absence of the crossing of the usual Binder parameter $g(L,T)$ will not imply the absence of the phase transition of this system. We speculate that, even when $D=0$, if the system becomes free from the drift, $g^{(\mathrm{diag})}(L,T)$ might exhibit a similar crossing behaviour.

Recently, it was proposed that the quantities $A(L,T)$ and $G(L,T)$ that measure the order-parameter fluctuations (OPF) exhibit crossing behaviour at T_{SG} even if $g(L,T)$ does not [31, 33, 34]

$$A(L,T)=\frac{[\langle q^2\rangle^2]-[\langle q^2\rangle]^2}{[\langle q^2\rangle]^2} \tag{10}$$

$$G(L,T)=\frac{[\langle q^2\rangle^2]-[\langle q^2\rangle]^2}{[\langle q^4\rangle]-[\langle q^2\rangle]^2}. \tag{11}$$

Then we also calculate $A(L,T)$ and $G(L,T)$ and examine their L-dependences. In figure 9, we show $G(L,T)$ for different L at $D=0$ and $D=0.2J$. When $D=0$, $G(L,T)$ for large L ($\geqslant 9$) seem to come together near T_{SG}. This property becomes more prominent in the anisotropic case of $D=0.2J$ where the data for smaller L ($=5,7$) join. We have also seen that $A(L,T)$ for both $D=0$ and $D\neq 0$ show a somewhat different crossing behaviour at a temperature a little higher than T_{SG}.

4. Conclusion

We have re-examined the spin-glass (SG) phase transition of the $\pm J$ Heisenberg models with and without random anisotropy D in three dimensions ($d=3$). Attention has been paid to the results of (i) the defect energy method and (ii) the Monte Carlo method, because the evidence of the absence of the SG phase transition at a finite temperature has been given by these two methods. Our results have been summarized as follows.

(i) The stiffness exponent θ. We have shown that the previous result of $\theta<0$ is not convincing for two reasons, i.e., (a) the meaning of the defect energy $[|\Delta E_L|]$ in the conventional method is not clear, and (b) even if the method is meaningful, the plot of $[|\Delta E_L|]$ as a function of L curves considerably. We have proposed a new method which considers the stiffness of the lattice itself. By using the method, we have shown that $\theta>0$ for $D\neq 0$ and the same is true for $D=0$. Having considered the stiffness at finite temperatures, we have obtained the SG phase transition temperature $T_{\mathrm{SG}}\sim 0.19J$ for $D=0$.

(ii) The Monte Carlo method. A large scale simulation has enabled us to make a scaling plot of the SG susceptibility χ_{SG} which suggests the finite transiton temperature of $T_{\mathrm{SG}}\sim 0.18J$ for $D=0$. The quantities $G(L,T)$ and $A(L,T)$ that measure the order-parameter fluctuations have exhibited a merging behaviour near T_{SG}, but the Binder parameter $g(L,T)$ has not exhibited the usual crossing behaviour. However, analyses of the model with $D\neq 0$ have suggested that the absence of the crossing of $g(L,T)$ will not mean the absence of the SG order in this model.

[5] We have also made a scaling plot of $\chi_{\mathrm{SG}}^{(\mathrm{diag})}$ and found that data are scaled fairly well using a similar transition temperature for both $D=0$ and $D\neq 0$.

Our results have confirmed that, in contrast to the common belief, the SG phase transition occurs at a finite temperature. It should be noted that the two different methods have given almost the same SG transition temperature of $T_{\rm SG} \sim 0.19J$. This value of the transition temperature is very close to that estimated from nonequilibrium properties of the model, i.e., $T_{\rm SG} \sim 0.19J$ from the ageing effect of the spin autocorrelation function [16] and $T_{\rm SG} \sim 0.21J$ from the nonequilibrium relaxation method [17]. Quite recently, Lee and Young [18] studied the Gaussian Heisenberg model, and suggested that $T_{\rm SG}/J = 0.16 \pm 0.02$. This value of the transition temperature is also reasonably close to our value, considering the difference of the bond distribution. Hence we conclude that the model exhibits the SG phase transition even when the anisotropy is absent and its transition temperature is $T_{\rm SG} \sim 0.19J$.

We make two comments. One might think that, for a larger D, the Ising values of the exponents ($\theta \sim 0.2$ [20], and $\nu \sim 2$ and $\eta = -0.3$ [35]) should be recovered. However, we consider that this opinion is not necessarily true, because the random anisotropy in the model of equation (1) is not uniaxial. Of course, we could not rule out the possibility that a finite-size effect masks true values. The other comment is that the strange behaviour of $g(L, T)$ of the Heisenberg SG model will come from the choice of the order parameter. When $D \neq 0$, the order parameter should be chosen as the sum of the diagonal components of the spin overlap, because only the inversion symmetry exists. We speculate that the same will be true in the isotropic case of $D = 0$, though the $O(3)$ symmetry recovers. To examine this speculation, we are currently making the simulation removing the uniform rotation of the system [16].

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